I. Advanced Studies

FUTURE PROJECTS

A. Lunar Elevation Correction for Gravity Measurements, *R. G. Brereton*[1]

The gravitational constant and the mass of the moon are both known to several significant figures. The ratio of the mass of the earth to that of the moon has been recently determined from radio tracking of space probes to be 81.303 (Ref. 1). Thus,

$$\frac{E}{M} = 81.303 \tag{1}$$

Given M, the mass of the earth, as 5.975×10^{27} g then the mass of the moon E can be calculated to be 7.349×10^{25} g. Using this value for the mass of the moon, a radius r of 1738 km, and a value for the gravitational constant G of 6.67×10^{-8} $cm^3/g\text{-}s^2$, then the acceleration of gravity g on the lunar surface is

$$g = G\frac{M}{r^2} = 162.3 \text{ cm/s (162.3 gal)} \tag{2}$$

[1]Assigned to *Apollo* Lunar Exploration Office, NASA, Washington, DC.

Errors in our knowledge of the lunar radius may affect this value by as much as ±100 mgal (1 gal = 1 cm/s^2 = 1000 mgal) and crustal anomalies by a like amount, but in general, if the shape of the lunar geoid is known and the value of the acceleration of gravity is known at one point, then the acceleration of lunar gravity can be computed for any other point. And conversely, if the value of the acceleration of gravity is known at all points on the lunar geoid, then the form of the level surface can be deduced.

Anomalies in the gravity field of the moon can be attributed to density contrasts in surface and subsurface rocks, to so-called terrain effects which lead to the free-air, Bouguer, and topographic corrections, and finally, shape effects. In order to use gravity data to interpret lunar structures and shape, it is necessary to make corrections for terrain effects on the data. In the following discussion, the various non-instrument corrections that will commonly be applied to lunar gravity data in order to relate all observations to a common datum are considered.

If the moon were a perfect fluid with no lateral variations in density, its surface would correspond to a

spheroid and the direction of gravity over this level surface would be everywhere perpendicular to it. Obviously, the lunar surface is not always parallel with a spheroid of reference and gravity measurements made on the actual surface will have to be corrected to the reference surface if gravity values are to be used for defining the shape of the moon or crustal structures. The largest corrections here will come from differences in elevation between observation points. Two corrections are required for this. They are the free-air and Bouguer corrections. The free-air correction takes into account the fact that gravity varies with elevation and measurements made at a higher elevation are farther from the center of mass and therefore have a smaller gravity value than measurements at a lower elevation. Since the moon's mass can be considered to be concentrated at its center, then if g_0 represents the value of gravity at its reference surface and g_h the value at height h, the inverse-square law tells us that the gravity difference between the two points is

$$\frac{g_0}{g_h} = \frac{(r+h)^2}{r^2} = \frac{r^2 + 2rh + h^2}{r^2} \tag{3}$$

and

$$\frac{g_0}{g_h} = 1 + \frac{2h}{r} + \frac{h^2}{r^2}$$

Because h will be small compared to r, the term h^2/r^2 can be neglected, and we can write

$$g_0 - g_h = \frac{2hg_h}{r} \tag{4}$$

This is also the same as the vertical gradient of gravity, where, from Eq. (2),

$$g = G\frac{M}{r^2} \text{ and } \frac{dg}{dr} = \frac{-2GM}{r^3} = \frac{-2g}{r} \tag{5}$$

Substituting,

$$\frac{dg}{dr} = -\frac{2(162.3) \times 10^3}{1738 \times 10^3} \text{ mgal/m}$$

$$= -0.187 \text{ mgal/m} \tag{6}$$

The lunar free-air correction adjusts gravity observations for points made at different elevations to a common datum, but it does not account for the change in mass under the station in order to change its elevation. In other words, to change the elevation of surface observation points requires that a specific volume of surface rock material is either added to or subtracted from the datum. This effect is called the Bouguer correction, which is always opposite in sign to the free-air correction. It is usual to assume either a slab or cylinder of infinite horizontal dimensions and finite vertical dimensions in order to compute an expression for the Bouguer correction. The value for the Bouguer correction is

$$\Delta B = 2\pi G\rho h = 0.042\rho \text{ mgal/m}$$

If we assume an average density ρ for lunar crustal rocks of say 2.67 g/cm^3, then the Bouguer correction is

$$0.112 \text{ mgal/m}$$

and the combined lunar elevation correction, that is, the free-air correction minus the Bouguer correction, is approximately 0.075 mgal/m. Interestingly, this value is less than one-half of the correction required on earth, and this has an important effect on gravimetric field operations on the moon since it reduces the needed accuracy for elevation control. It should be realized that the magnitude of the Bouguer correction will change for sedimentary areas in contrast to igneous areas, and also in response to regional investigations in contrast to local studies. The Bouguer correction is very sensitive to rock densities and in a local survey in hilly country it can serve as a precise tool for determining the average density of local rock units. Table 1 gives a range of lunar elevation corrections as a function of density.

Table 1. Lunar elevation correction s

Free-air correction, mgal/m	Density ρ g/cm^3	Bouguer correction, mgal/m	Elevation correction, mgal/m
0.187	1.5	0.063	0.124
0.187	2.0	0.084	0.103
0.187	2.5	0.105	0.082
0.187	3.0	0.126	0.061

A topographic correction will only be required in hilly areas. It may be thought of as a supplemental correction to the smooth infinite slab hypothesized in making the Bouguer correction, for it accounts for all material projecting above the Bouguer plain and all material needed to fill in hollows below the plain. It is always positive and, for most survey work, it will not be troublesome.

Tidal corrections for gravity measurements on the lunar surface, caused by the differential attraction of the moon by the earth as the former moves between perigee and apogee, may be larger than expected if the crust of the moon yields to the external force. The external force difference between perigee and apogee is very small, of the order of a milligal, but if the lunar crust yields to this force, then gravity readings on the surface will be affected. It is not likely that this effect will be significant for mobile surface exploration since the effect probably amounts to less than a milligal per lunation.

The scientific objectives and instruments for lunar gravity investigations were discussed in SPS 37-57, Vol. III, pp. 1–6, and SPS 37-62, Vol. III, pp. 1–5.

Reference

1. Melbourne, W. G., "The Determination of Planetary Masses From Radio Tracking of Space Probes and Planetary Radar," *Symposium C Space Probes, Part I,* presented at the 12th Plenary Meeting of COSPAR, Prague, Czechoslovakia, May 11–24, 1969.